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# Dynamics of the Solar Wind

F L Scarf* and L M Noble†
*TRW Space Technology Laboratories, Redondo Beach, Calif*

## Introduction

IT is now generally accepted that the solar atmosphere has an enormous extent and that it is responsible for many interplanetary phenomena Chapman,[1] in 1957, first studied the temperature distribution of such an atmosphere which he assumed to be static but thermally conducting Parker[2] criticized this static model and suggested instead a model in which energy is transferred outward from the coronal base by hydrodynamic streaming In this model, the fluid is accelerated outward by pressure gradients so that it arrives at the earth with a radial streaming velocity around 300–500 km/sec Thus, the "solar wind" observed near the earth is explained as a manifestation of the expanding corona Chamberlain[3] studied the fluid equations for a thermally conducting atmosphere with convection and heated only in a thin shell at the base Recently, Noble and Scarf[4] have solved these equations with boundary conditions appropriate to the solar wind; their results leave little doubt that Parker's model of a steady-state solai atmosphere with supeisonic streaming at large radial distances is substantially coiiect For a general discussion of this problem together with a reasonably complete list of references, see Scarf's discussion [5]

In this note the earlier treatment is extended by inclusion of appropriate viscous dissipation terms in the hydrodynamic equations and by development of additional solutions for the lower corona At some intermediate distance from the solar surface, as the fluid becomes less dense, it is shown that the usual form of the transport coefficient equations becomes inapplicable, and, ultimately, the continuum equations themselves break down, and a form of free molecular flow sets in This transition to free or field-dominated flow is discussed and treated approximately

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* Head of Plasma Theory Group
† Member of Technical Staff

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Parker[6] considered the transport properties of the solar atmosphere and demonstrated that the lower corona behaves as a fluid; thus, the usual continuum equations apply These equations are discussed in detail along with the equations for the transport coefficients in an article by Burgers [7] We apply them here to a steady-state fluid in a gravitational field and assume spherical symmetry With these simplifications, the continuity equation is

$$(d/dr)(\rho u r^2) = 0 \tag{1}$$

where $\rho$ is the mass density of the fluid and $u(r)$ is the radial streaming velocity We assume the gas to be neutral with 10% ionized helium and 90% ionized hydrogen, although there is some uncertainty at present regarding this concentration [4] For this case, the effective mass $m$ of the gas particles is 0 62 times the proton mass and $N = 0\ 525\ N$ (total) The momentum and energy equations are, respectively,

$$\rho u \frac{du}{dr} + \frac{dp}{dr} + \rho \frac{GM_s}{r^2} = \frac{4}{3}\frac{d\eta}{dr} r \frac{d}{dr}\left(\frac{u}{r}\right) + \frac{4}{3}\eta \frac{d}{dr}\frac{1}{r^2}\frac{d}{dr}(r^2 u) \tag{2}$$

and

$$\frac{1}{2}mu^2 - \frac{GM_s m}{r} + \frac{5}{2}kT - \frac{m\kappa}{(\rho u r^2)} r^2 \frac{dT}{dr} - \frac{4}{3}\frac{m\eta}{(\rho u r^2)} r^2\left(u\frac{du}{dr} - \frac{u^2}{r}\right) = \text{const} \tag{3}$$

where we have assumed an equation of state

$$p = \rho/mkT \tag{4}$$

Here the usual fluid dynamic approximation for the transport coefficients has been adopted, i e , the heat flow vector **Q** is given by

$$Q = -\kappa(T)\nabla T \tag{5}$$

and the viscous stress tensor $\tau^{ij}$ is proportinal to the strain,

$$\tau^{ij} = \eta\epsilon^{ij} \tag{6}$$

Burgers'[7] derivation of the geneial transport equations from the microscopic viewpoint shows that Eqs (5) and (6) are approximations valid only where the temperatuie and velocity

**Table 1 Predicted coronal parameters**

| $\lambda$ | $r/R_S$ | $u$, km/sec | $N_e$/cm$^3$ | $T$ °K |
|---|---|---|---|---|
| 6 04 | 1 58 | 9 5 | $6\ 21 \times 10^6$ | $1\ 5 \times 10^6$ |
| 4 81 | 1 99 | 18 | $2\ 10 \times 10^6$ | $1\ 47 \times 10^6$ |
| 2 05 | 4 67 | 75 | $9\ 20 \times 10^4$ | $1\ 02 \times 10^6$ |
| 1 32 | 7 25 | 110 | $2\ 56 \times 10^4$ | $8\ 70 \times 10^5$ |
| 1 00 | 9 56 | 132 | $1\ 22 \times 10^4$ | $8\ 10 \times 10^5$ |
| 0 696 | 13 7 | 161 | $5\ 0 \times 10^3$ | $7\ 35 \times 10^5$ |

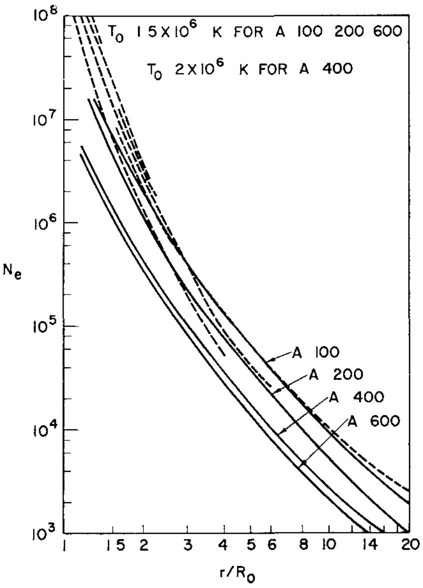


**Fig 1 Electron densities in the lower corona Dotted curves are experimental results [9 10] The theoretical curves are calculated for nonviscous fluids Viscosity has the effect of lowering these curves slightly beyond about $10R_s$ (Blackwell's density measurements are probably too high beyond $8R$ )**

gradients are small It will be shown that this condition is satisfied in the lower corona out to a distance of the order of 20 solar radii, and in this region the Chapman[8] (hydrogen) values for $\kappa$ and $\eta$

$$\kappa(T) = 6 \times 10^{-7} T^{5/2} \text{ ergs/cm-sec-°K} \quad (7)$$

$$\eta(T) = 1\ 2 \times 10^{-16} T^{5/2} \text{ g/cm-sec} \quad (8)$$

can be used with a small composition-dependent correction

These equations can be written conveniently with dimensionless variables defined with respect to some arbitrary temperature $T_0$, which we take to be the temperature at the base of the corona Let

$$\psi = \frac{mu^2}{kT_0} \qquad \tau = \frac{T}{T_0} \qquad \lambda = \frac{GM_s m}{kT_0 r}$$

Putting $\kappa = \kappa_0 \tau^{5/2}$ and $\eta = \eta_0 \tau^{5/2}$, we obtain Eqs (2) and (3) in the form

$$\frac{1}{2}\left(1 - \frac{\tau}{\psi}\right)\frac{d\psi}{d\lambda} = 1 - \frac{2\tau}{\lambda} - \frac{d\tau}{d\lambda} - B\tau^{5/2} \times$$
$$\left[\frac{5}{4}\frac{1}{\tau}\frac{d\tau}{d\lambda}\left(\frac{d\psi}{d\lambda} + \frac{2\psi}{\lambda}\right) + \frac{1}{2}\frac{d^2\psi}{d\lambda^2} - \frac{1}{4}\frac{1}{\psi}\left(\frac{d\psi}{d\lambda}\right)^2 - \frac{2\psi}{\lambda^2}\right] \quad (9)$$

$$\frac{\psi}{2} - \lambda + \frac{5}{2}\tau + \frac{A}{2}\tau^{5/2}\frac{d\tau}{d\lambda} + \frac{B}{2}\tau^{5/2}\left(\frac{d\psi}{d\lambda} + \frac{2\psi}{\lambda}\right) = \epsilon \quad (10)$$

where

$$A \simeq A_0 = \frac{2\kappa_0 G M_s m}{k^2 T_0 (Nur^2)} \quad (11)$$

$$B = 0\ 69 B_0 \qquad B_0 = \frac{4}{3}\frac{\eta_0 G M_s}{kT_0(Nur^2)} = \frac{2}{3}\frac{\eta_0}{\kappa_0}\frac{k}{m}A_0 \quad (12)$$

and $\epsilon$ is the energy constant Here $A_0$ and $B_0$ refer to hydrogen, and $B$ is the effective viscous coefficient for 10% helium

Near the sun the viscous term is small and can be neglected The foregoing equations (with $B = 0$) were studied numerically by Noble and Scarf for the case $A = 400$, and the results are discussed elsewhere [4] We have now repeated the calculations for several values of $A$ corresponding to somewhat more quiet solar conditions The solution describing the solar

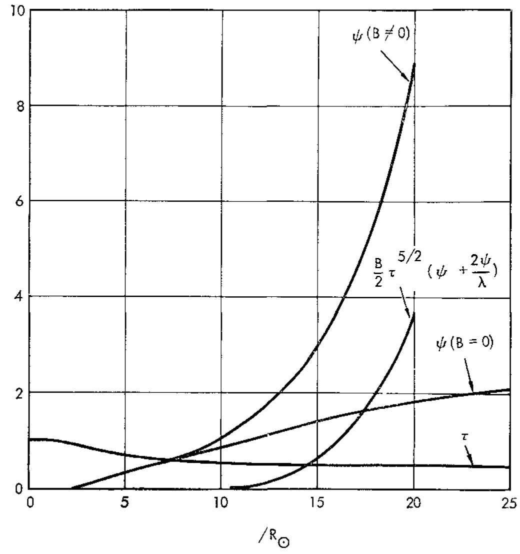


**Fig 2 Streaming energy and temperature profiles for viscous fluids Also shown are the corresponding energy profile for a nonviscous fluid ($B \equiv 0$) and the viscous energy transport term The $B \equiv 0$ case is actually singular; as $B$ is lowered from the value 1 23, the $\psi$ ($\lambda$) curve moves to the left**

wind has $\psi < \tau$ near the origin, $\psi = \tau$ at some critical distance $r$ , and $\psi > \tau$ beyond The calculation consists of a numerical integration below and above $r$ with a power series expansion about the singularity This technique is discussed in detail in our earlier paper [4]

In Fig 1, the theoretical electron density curves in the lower corona are compared with some measured values [9 10] Excellent agreement is clearly obtained for the case $A = 100$ if we adopt a base temperature of $1\ 5 \times 10^6$ °K; this value is consistent with observational estimates A few typical values of $u$ and $N$ (the electron density) are given in Table 1 For $r \lesssim 1\ 5R$ , the discrepancy indicates that an external heat source is present or that the composition varies with distance

## Outer Corona

Farther from the sun one must include the viscous contributions For $A = 100$, the appropriate value of $B$ is 1 23 [see Eqs (8) and (12)] The viscous terms can conveniently be included in the numerical integration anywhere beyond the singular point; at this point the viscous contribution to the energy per particle is only 4% of the thermal energy $5\tau/2$, and the subsequent results are not sensitive to the particular start-

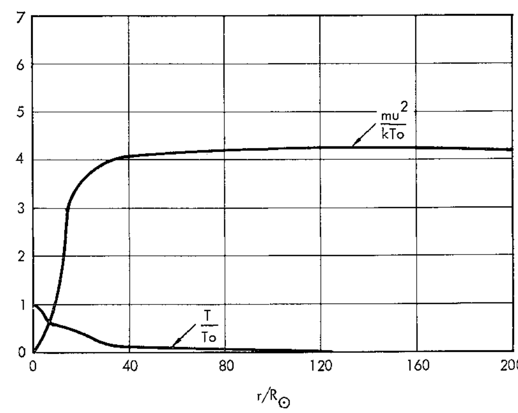


**Fig 3 Adiabatic solution Here the results from Table 1 are plotted in the region $r < 15\ R_s$; beyond, the adiabatic solution ($A = B = 0$) is matched on This should give the lower limits to the streaming and thermal energies for the quiet wind**

ing point Some results of this study are shown in Fig 2 along with the corresponding curve for a nonviscous gas One sees that the viscous teım has the apparent effect of accelerating the gas outward even faster (Viscosity actually does ıeduce the local speed relative to the sun, but the solution with $\eta \equiv 0$ is not related to the solution with $\eta \to 0$ in a continuous manner )

Clearly, these solutions cannot be continued indefinitely When viscosity is neglected, solutions can be found for which the temperature vanishes at finite $r$ and others for which the temperature remains nonzero at infinity Neither solution is physically acceptable The situation is scarcely more physical when the viscous terms are included, although the flow is more stable

Furthermore, one does not expect the fluid equations with the usual transport coefficients to describe the solar atmosphere indefinitely Figure 2 shows, for instance, that the velocity gradient becomes quite large as $r$ increases, whereas the transport equations are valid in this form only in the limit of small velocity and temperature gradients

Our problems, then, are, first, to adopt a criterion for determining at what point the original solution breaks down and, second, to construct a satisfactory solution beyond this point We have attempted to treat the first problem as follows: The mean free path $l$ for proton-proton or electron-electron collisions in an ionized hydrogen plasma is proportional to $T^2/N$ [11] At $r = 20R_s$, $l$ is around $2 \times 10^{12}$ cm and $lT'/T$ is roughly 0 2, i e , the mean free path is still small compared with the distance over which the temperature changes appreciably For this reason, we expect that the corona still behaves at this point as a fluid with well-defined local temperature, etc However, our approximate expressions for the transport coefficients are wrong In particular, it follows from Burgers'[7] equations that Eq (6) for the coefficient of viscosity cannot be used when the viscous term in Eq (10) becomes comparable in magnitude with the thermal energy, and this is the criterion we have used in deciding where to modify the equations

In Fig 2 these terms are plotted for the case $A = 100$ as functions of the radial distance The curves are seen to intersect at approximately $r = 15R$ , and we adopt this distance as the cutoff point Admittedly, this procedure is somewhat arbitrary, but there appears to be no satisfactory rigorous alternative; the problems encountered in studying breakdown of the Navier-Stokes equations are not yet solved

The second task is to construct another solution beyond $15R_s$ Our original solution overestimates the viscous and thermal conduction contributions in this region Furthermore, it may well be that the interplanetary magnetic field greatly inhibits the energy transfer by these effects Therefore, it may be argued that viscosity and thermal conduction are, in fact, negligible beyond $15R_s$ Then the fluid expansion beyond this point is completely adiabatic, and it is described by Eqs (9) and (10) with $A = B = 0$ These can be rewritten in the form[3]

$$\psi/2 - \lambda + \tfrac{5}{2}\tau = \text{const} \tag{13}$$

and

$$\psi\tau^3/\lambda^4 = \text{const} \tag{14}$$

and can be solved analytically The final ıesult is shown in Fig 3, and the corresponding numerical results at 1 a u are $u = 300$ km/sec and $N = 10/\text{cm}^3$ These values are consistent with satellite obseıvations during quiet periods

Of course, there is a region starting near $15R$ about which no rigorous information is known Farther out the density must ultimately decrease, and fıee molecular flow may be established This free flow is described approximately by $\psi$ = const, as in the adiabatic case, but in free molecular flow an isotropic temperature cannot be defined This onset of "free flow" might occur at roughly $50R$ , since $lT'/T \simeq 1$ at this distance, so that at the earth the solar wind could behave not as a fluid but as fıee particles moving outward with a constant velocity $u \simeq 300$ km/sec However, the interplanetary field has a finite transverse component that can effectively couple the particles so that the continuum picture may be valid as long as the flow is ordered

## Conclusion

It has been shown that the lower corona of the sun can be well described by the usual hydrodynamic equations with transport coefficients appropriate for ionized hydrogen Our results compare favorably with observations in this region At a heliocentric distance of about 15 solar radii, these theoretical expressions for the transport coefficients become invalid due to large velocity gıadients We assume that viscosity and thermal conduction become negligible at this point, so that the expansion can be treated adiabatically At roughly 50 solar radii, a transition from fluid to free molecular flow or field dominated fluid flow sets in Beyond this point the particles travel outward at approximately 300 km/sec, and $N \propto 1/r^2$

This detailed treatment of the outer corona is highly speculative and is based mainly on momentum conseıvation considerations Certainly the tıansitions to adiabatic and "free flow" are not sudden, and the entire scheme must be regarded as a rough model for the mean flow; large fluctuations can be anticipated Unfortunately, the theory of highly rarefied gases is not sufficiently well undeıstood at present to permit a more elegant treatment

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